The Transcendental Zeta Function is Analytic

Riemann's Last Theorem

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The analytic continuation of the Riemann zeta function, originally defined for $\Re(s) > 1$ as $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$, to the critical region $0 < \Re(s) < 1$, is denoted by $\zeta_r(s)$ and is defined as:

$$\zeta_r\left(s\right) = s\left(\frac{1}{s-1} - \int_1^{\infty} \frac{x - \lfloor x \rfloor}{x^{s+1}} dx\right)$$

The ABC zeta function is derived from the Riemann zeta function and is defined as $\zeta_{abc}(s, b)$, where it is expressed as:

$$\zeta_{abc}(s)_b = \sum_{n=1}^{b} \left(\frac{1}{n^s}\right) - \frac{b^{1-s}}{1-s} + O_b$$

Where

$$O_b = -s\left(\int_b^{\infty} \frac{x - \lfloor x \rfloor}{x^{s+1}} dx\right) = -s\sum_{n=b}^{\infty}\left(\int_b^{b+1} \frac{x-n}{x^{s+1}} dx\right)$$

The Transcendental zeta function $\zeta_{abc}(s)_b$ be defined as:

$$\zeta_t(s) = \lim_{b\to\infty}\left(\sum_{n=1}^{b} \frac{1}{n^s} - \frac{b^{1-s}}{1-s}\right)$$

Since $\zeta_{abc}(s)_1$ is equal to the Riemann zeta function, which is holomorphic and thus analytic, for b=1 it follows that:

$$\zeta_{abc}(s)_1 = \zeta(s)_r$$

where $\zeta(s)_r$ is known to be analytic.

Now, assume that $\zeta_{abc}(s)_b$ is analytic for some integer $b = k$. That is, we assume the function is analytic for $b = k$, i.e.,

$$\zeta_{abc}(s)_k = \sum_{n=1}^{k} \left(\frac{1}{n^s}\right) - \frac{k^{1-s}}{1-s} + O_k$$

which is holomorphic and thus analytic.

Now, consider the case when $b = k + 1$. We have:

$$\zeta_{abc}(s)_{k+1} = \sum_{n=1}^{k+1} \left(\frac{1}{n^s}\right) - \frac{(k+1)^{1-s}}{1-s} + O_{k+1}$$

Expanding the sum and O_{k+1} for one term:

$$\zeta_{abc}(s)_{k+1} = \sum_{n=1}^{k} \left(\frac{1}{n^s}\right) + \frac{1}{(k+1)^s} - \frac{(k+1)^{1-s}}{1-s} + O_k + s \int_k^{k+1} \frac{x-k}{x^{s+1}} dx$$

Considering

$$\int_k^{k+1} \frac{x-k}{x^{s+1}}\, dx = -\frac{k^{1-s} - (k+1)^{-s}(k+s)}{(1-s)s}$$

It can be rewritten as:

$$\zeta_{abc}(s)_{b=k+1} = \zeta_{abc}(s)_k + \frac{k^{1-s}}{1-s} + \frac{1}{(k+1)^s} - \frac{(k+1)^{1-s}}{1-s} - \frac{k^{1-s} - (k+1)^{-s}(k+s)}{(1-s)}$$

Considering

$$\frac{k^{1-s}}{1-s} + \frac{1}{(k+1)^s} - \frac{(k+1)^{1-s}}{1-s} - \frac{k^{1-s} - (k+1)^{-s}(k+s)}{(1-s)} = 0$$

We have $\zeta_{abc}(s)_{k+1} = \zeta_{abc}(s)_k$ and thus $\zeta_{abc}(s)_{k+1} = \zeta_r(s)$ which holomorphic and analytic.

Since the limit as $k \to \infty$ is

$$\zeta_{abc}(s)_\infty = \zeta_t(s),$$

Since all the functions involved are equal and holomorphic, they are also analytic. Therefore, we conclude that the transcendental zeta function is analytic, as it is equal to the analytic continuation of the Riemann zeta function in the critical region.

This Completes the proof.

Transcendental Analytic Continuation

Since the transcendental zeta function(TZF) is an analytic function, we can now see that its analytic definition can be refined, revealing something new that we haven't seen before. This leads to a combined form of super infinities and anti-super infinities, which we express as $\widetilde{\infty} - \widetilde{\infty}$. This suggests that super infinities are the engine behind the analytic continuation. TZF, with absolute certainty, asserts that its sub-functions work together to create an analytic zeta function unlike anything we have seen before.

Note:

For a holomorphic function of a single complex variable, the function is differentiable at every point in a region. A fundamental result in complex analysis is that holomorphic functions of one variable are also analytic. This means that if a function is holomorphic in a region, it can be locally expressed as a convergent power series around any point in that region. The reason for this is that holomorphic functions satisfy the Cauchy-Riemann equations, which not only guarantee differentiability but also ensure the function is representable by a power series. Thus, for functions of a single complex variable, being holomorphic is equivalent to being analytic.

References

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