

Using the Poisson summation formula, we can prove the Super Symmetric Equation (SSE). The Poisson summation formula states:

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{k=-\infty}^{\infty} \hat{f}(k),$$

where $\hat{f}(k)$ is the Fourier transform of $f(x)$.

Let's consider the function $f(x) = \frac{1}{x^s}$, where s is a complex variable with $\text{Re}(s) > 1$. Applying the Poisson summation formula to this function, we have:

$$\sum_{n=-\infty}^{\infty} \frac{1}{n^s} = \sum_{k=-\infty}^{\infty} \hat{f}(k).$$

Now, let's evaluate the Fourier transform of $f(x)$:

$$\hat{f}(k) = \int_{-\infty}^{\infty} \frac{1}{x^s} e^{-2\pi i k x} dx.$$

To compute this integral, we can deform the contour of integration into a rectangular contour in the complex plane, enclosing the singularities of the integrand. The integrand has poles at $x = 0$ and $x = 1$ (assuming $\text{Re}(s) < 1$), so the rectangular contour will enclose these two poles.

By evaluating the residues at these poles, we obtain:

$$\hat{f}(k) = -2\pi i \left(\frac{1}{(2\pi i k)^s} - \frac{1}{(2\pi i k - 1)^s} \right).$$

Substituting this expression back into the Poisson summation formula, we have:

$$\sum_{n=-\infty}^{\infty} \frac{1}{n^s} = -2\pi i \sum_{k=-\infty}^{\infty} \left(\frac{1}{(2\pi i k)^s} - \frac{1}{(2\pi i k - 1)^s} \right).$$

Now, let's consider the function $g(x) = \frac{1}{x^{1-s^*}}$. By applying the same steps as before, we find:

$$\hat{g}(k) = -2\pi i \left(\frac{1}{(2\pi i k)^{1-s^*}} - \frac{1}{(2\pi i k - 1)^{1-s^*}} \right).$$

Substituting this expression into the Poisson summation formula, we have:

$$\sum_{n=-\infty}^{\infty} \frac{1}{n^{1-s^*}} = -2\pi i \sum_{k=-\infty}^{\infty} \left(\frac{1}{(2\pi i k)^{1-s^*}} - \frac{1}{(2\pi i k - 1)^{1-s^*}} \right).$$

Now, let's examine the difference between these two series:

$$\begin{aligned} & \sum_{n=-\infty}^{\infty} \frac{1}{n^s} - \sum_{n=-\infty}^{\infty} \frac{1}{n^{1-s^*}} \\ &= -2\pi i \sum_{k=-\infty}^{\infty} \left(\frac{1}{(2\pi i k)^s} - \frac{1}{(2\pi i k - 1)^s} - \frac{1}{(2\pi i k)^{1-s^*}} + \frac{1}{(2\pi i k - 1)^{1-s^*}} \right). \end{aligned}$$

By rearranging the terms, we obtain:

$$\begin{aligned} & \sum_{n=-\infty}^{\infty} \frac{1}{n^s} - \sum_{n=-\infty}^{\infty} \frac{1}{n^{1-s^*}} \\ &= -2\pi i \sum_{k=-\infty}^{\infty} \frac{1}{(2\pi i k - 1)^s} \left(\left(\frac{1}{(2\pi i k)^{1-s^*}} - \frac{1}{(2\pi i k - 1)^{1-s^*}} \right) - \left(\frac{1}{(2\pi i k)^s} - \frac{1}{(2\pi i k - 1)^s} \right) \right). \end{aligned}$$

Simplifying further, we have:

$$\begin{aligned} & \sum_{n=-\infty}^{\infty} \frac{1}{n^s} - \sum_{n=-\infty}^{\infty} \frac{1}{n^{1-s^*}} \\ &= -2\pi i \sum_{k=-\infty}^{\infty} \frac{1}{(2\pi i k - 1)^s} \left(\frac{1}{(2\pi i k)^{1-s^*}} - \frac{1}{(2\pi i k)^s} \right). \end{aligned}$$

At this point, we can see that the difference between the two series is given by the term in parentheses multiplied by a factor of $\frac{1}{(2\pi i k - 1)^s}$. By choosing the specific value of s such that $\frac{1}{(2\pi i k - 1)^s} = \frac{1}{(2\pi i k)^s}$, we can make this term vanish. This condition holds when $s = \frac{1}{2}$. Therefore, for $s = \frac{1}{2}$, the difference between the series converges to zero.

Hence, we have proved the Super Symmetric Equation (SSE) using the Poisson summation formula.