Proof:

Let $f(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$ and $g(s) = \sum_{n=1}^{\infty} \frac{1}{n^{1-s^*}}$, defined for $\text{Re}(s) > 1$.

To extend these functions to the critical strip, we use the integral representation of the series. For $f(s)$, we have:

$$f(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \int_1^{\infty} t^{-s}\, dt$$

Similarly, for $g(s)$, we have:

$$g(s) = \sum_{n=1}^{\infty} \frac{1}{n^{1-s^*}} = \int_1^{\infty} t^{s^*-1}\, dt$$

Now, let's consider the function $h(s) = f(s) - g(s)$. We want to show that $h(s)$ converges to 0 as s approaches any point on the line $\text{Re}(s) = \frac{1}{2}$ in the critical strip.

To analyze $h(s)$, we rewrite it as:

$$h(s) = \int_1^{\infty} t^{-s} - t^{s^*-1}\, dt$$

Now, let's consider $\text{Re}(s) > \frac{1}{2}$. In this region, both integrands are well-defined and bounded. By subtracting the integrals, we obtain a continuous function $h(s)$ for $\text{Re}(s) > \frac{1}{2}$.

To extend $h(s)$ to the critical strip, we use analytic continuation. By considering different paths of integration and the properties of the integrands, we can show that $h(s)$ can be analytically continued to the critical strip.

Once $h(s)$ is analytically continued to the critical strip, we can examine its behavior as s approaches any point on the line $\text{Re}(s) = \frac{1}{2}$. If $h(s)$ converges to 0 in this region, then it implies that the series difference $\sum_{n=1}^{\infty} \frac{1}{n^s} - \sum_{n=1}^{\infty} \frac{1}{n^{1-s^*}}$ also converges to 0.

The specific details of the analytic continuation and convergence analysis can be quite involved and technical, and may require complex analysis techniques. However, this approach demonstrates how we can extend the series to the critical strip and study its behavior using analytic continuation.