1 The Supersymmetric Equation

The supersymmetric equation is a mathematical equation that relates the values of the Riemann zeta function at two different points. The equation states that if $\zeta(s) = \zeta(1 - s^*)$, then the difference between the sums of the series $\sum_{n=1}^{\infty} \frac{1}{n^s}$ and $\sum_{n=1}^{\infty} \frac{1}{n^{1-s^*}}$ converges to zero in the critical strip. The converse is also true.

The critical strip is the region of the complex plane $s \in C : 0 < Re(s) < 1$. The Riemann zeta function is defined for all complex numbers with $Re(s) > 1$, but it can be analytically continued to the critical strip. The supersymmetric equation is a consequence of the analytic continuation of the Riemann zeta function.

2 The Identity Theorem

The identity theorem is a theorem in complex analysis that states that if two holomorphic functions are equal on an open set, then they are equal everywhere.

[Identity Theorem]

Let U be an open set in C, and let f and g be holomorphic functions on U. If $f(z) = g(z)$ for all $z \in U$, then $f(z) = g(z)$ for all $z \in C$.

3 Proof of the Supersymmetric Equation

Let $f(s) = \zeta(s) - \zeta(1 - s^*)$. Then $f(s)$ is holomorphic on the critical strip. This is because the Riemann zeta function is holomorphic on the critical strip.

We know that $f(s) = 0$ on the line $Re(s) = \frac{1}{2}$.

The identity theorem tells us that $f(s) = 0$ everywhere on the critical strip. This is equivalent to the supersymmetric equation.

Let s_0 be a point on the line $Re(s) = \frac{1}{2}$. Then $f(s_0) = 0$.

The identity theorem tells us that if $f(s_0) = 0$ for some $s_0 \in U$, then $f(s) = 0$ for all $s \in U$.

Therefore, $f(s) = 0$ for all $s \in U$. This is equivalent to the supersymmetric equation.